1 (i)	$\overrightarrow{AC}\times\overrightarrow{AB}=\begin{pmatrix}5\\-8\\-26\end{pmatrix}\times\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}42\\-42\\21\end{pmatrix}$ Perpendicular distance is $\dfrac{\lvert\overrightarrow{AC}\times\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}$ $=\dfrac{\sqrt{42^2+42^2+21^2}}{\sqrt{2^2+1^2+2^2}}=\dfrac{63}{3}$ $=21$	B2 M1 M1 A1 **5**	Give B1 for one component correct Calculating magnitude of a vector product www
	OR $\left[\begin{pmatrix}3+2\lambda\\8+\lambda\\27-2\lambda\end{pmatrix}-\begin{pmatrix}8\\0\\1\end{pmatrix}\right].\begin{pmatrix}2\\1\\-2\end{pmatrix}=0$ M1 A1 $2(2\lambda-5)+(\lambda+8)-2(-2\lambda+26)=0$ A1 ft $\lambda=6$ [F is $(15,\ 14,\ 15)$] $\text{CF}=\sqrt{7^2+14^2+14^2}=21$ M1A1		Appropriate scalar product
(ii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}2\\1\\-2\end{pmatrix}\times\begin{pmatrix}3\\p\\p-1\end{pmatrix}=\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$ $\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})=\begin{pmatrix}5\\-8\\-26\end{pmatrix}.\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$ $=5(3p-1)-8(-2p-4)-26(2p-3)$ [$=-21p+105$] $\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert=\sqrt{(3p-1)^2+(-2p-4)^2+(2p-3)^2}$ $=\sqrt{17p^2-2p+26}$ Distance is $\dfrac{\lvert\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})\rvert}{\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert}=\dfrac{21\lvert p-5\rvert}{\sqrt{17p^2-2p+26}}$	B1 B1 B1 M1 A1 ft B1 ft M1A1 (ag) **8**	 Correctly obtained
(iii)	$V=(\pm)\ \frac{1}{6}(\overrightarrow{AC}\times\overrightarrow{AB}).\overrightarrow{AD}=(\pm)\ \frac{1}{6}\begin{pmatrix}42\\-42\\21\end{pmatrix}.\begin{pmatrix}8\\p-8\\p-27\end{pmatrix}$ $=(\pm)\ 56-7(p-8)+\frac{7}{2}(p-27)$ $=(\pm)\ \frac{35}{2}-\frac{7}{2}p$ $=\frac{7}{2}\lvert p-5\rvert$	M1 A1 ft M1 A1 **4**	Appropriate scalar triple product In any form Evaluation of scalar triple product *Dependent on previous M1* $\frac{1}{6}(105-21p)$ or better
(iv)	Intersect when $p=5$ $\begin{pmatrix}3\\8\\27\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}8\\0\\1\end{pmatrix}+\mu\begin{pmatrix}3\\5\\4\end{pmatrix}$ $3+2\lambda=8+3\mu$ $8+\lambda=5\mu$ [$8+\lambda=p\mu$] $27-2\lambda=1+4\mu$ [$27-2\lambda=1+(p-1)\mu$] $\lambda=7,\ \ \mu=3$ Point of intersection is $(17,\ 15,\ 13)$	B1 B1 ft M1 A1 ft A1 ft M1 A1 **7**	 Equations of both lines *(may involve p)* Equation for intersection *(must have different parameters)* Equation involving λ and μ Second equation involving λ and μ *or* Two equations in $\lambda,\ \mu,\ p$ Obtaining λ or μ

2 (i)	$\dfrac{\partial g}{\partial x}=(y+xy+z^2)\,e^{x-2y}$ $\dfrac{\partial g}{\partial y}=(x-2xy-2z^2)\,e^{x-2y}$ $\dfrac{\partial g}{\partial z}=2z\,e^{x-2y}$	M1 A1 A1 A1 **4**	Partial differentiation
(ii)	At $(2,1,-1)$, $\dfrac{\partial g}{\partial x}=4$, $\dfrac{\partial g}{\partial y}=-4$, $\dfrac{\partial g}{\partial z}=-2$ Normal has direction $\begin{pmatrix}4\\-4\\-2\end{pmatrix}$ L passes through $(2,1,-1)$ and has this direction	M1 A1 M1 A1 (ag) **4**	
(iii)	When $g=0$, $xy+z^2=0$ $(2-2\lambda)(1+2\lambda)+(-1+\lambda)^2=0$ $3-3\lambda^2=0$ $\lambda=\pm1$ $\lambda=1$ gives $P(0,3,0)$ $\lambda=-1$ gives $Q(4,-1,-2)$	M1 M1 A1 (ag) A1 **4**	Obtaining a value of λ Or B1 for verifying $g(0,3,0)=0$ *and* showing that P is on L
(iv)	At P, $\dfrac{\partial g}{\partial x}=3e^{-6}$, $\dfrac{\partial g}{\partial y}=0$, $\dfrac{\partial g}{\partial z}=0$ $\delta g\approx\dfrac{\partial g}{\partial x}\delta x+\dfrac{\partial g}{\partial y}\delta y+\dfrac{\partial g}{\partial z}\delta z$ $=3e^{-6}(-2\mu)+0+0=-6\mu e^{-6}$	M1 M1 A1 (ag) **3**	OR give M2 A1 www for $g(-2\mu,\ 3+2\mu,\ \mu)$ $=(-3\mu^2-6\mu)\,e^{-6\mu-6}\approx-6\mu\,e^{-6}$
(v)	When $-6\mu\,e^{-6}\approx h$, $\mu\approx-\frac{1}{6}e^6h$ Point $(-2\mu,\ 3+2\mu,\ \mu)$ is approximately $(\frac{1}{3}e^6h,\ 3-\frac{1}{3}e^6h,\ -\frac{1}{6}e^6h)$	M1 A1 (ag) **2**	
(vi)	At Q, $\dfrac{\partial g}{\partial x}=-e^6$, $\dfrac{\partial g}{\partial y}=4e^6$, $\dfrac{\partial g}{\partial z}=-4e^6$ When $x=4-2\mu$, $y=-1+2\mu$, $z=-2+\mu$ $\delta g\approx(-e^6)(-2\mu)+(4e^6)(2\mu)+(-4e^6)(\mu)$ $=6\mu\,e^6$ If $6\mu\,e^6\approx h$, then $\mu\approx\frac{1}{6}e^{-6}h$ Point is approximately $(4-\frac{1}{3}e^{-6}h,\ -1+\frac{1}{3}e^{-6}h,\ -2+\frac{1}{6}e^{-6}h)$	M1 M1 M1A1 M1 A2 **7**	OR give M1 M2 A1 www for $g(4-2\mu,\ -1+2\mu,\ -2+\mu)$ $=(-3\mu^2+6\mu)\,e^{-6\mu+6}\approx6\mu\,e^6$ Give A1 for one coordinate correct *If partial derivatives are not evaluated at Q, max mark is M0M1M0M0*

3 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$	B1	
	$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1 + (\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2})^2$	M1	
	$= 1 + \frac{1}{4}x^{-1} - \frac{1}{2} + \frac{1}{4}x = \frac{1}{4}x^{-1} + \frac{1}{2} + \frac{1}{4}x$		
	$= (\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2})^2$	A1	
	Arc length is $\int_0^a (\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2})\,\mathrm{d}x$		
	$= \left[x^{1/2} + \frac{1}{3}x^{3/2} \right]_0^a$	M1	
	$= a^{1/2} + \frac{1}{3}a^{3/2}$	A1 (ag) **5**	
(ii)	Curved surface area is $\int 2\pi y\,\mathrm{d}s$	M1	For $\int y\,\mathrm{d}s$
	$= \int_0^3 2\pi(x^{1/2} - \frac{1}{3}x^{3/2})(\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2})\,\mathrm{d}x$	A1	Correct integral form *including limits*
	$= 2\pi\int_0^3 (\frac{1}{2} + \frac{1}{3}x - \frac{1}{6}x^2)\,\mathrm{d}x$		
	$= 2\pi\left[\frac{1}{2}x + \frac{1}{6}x^2 - \frac{1}{18}x^3 \right]_0^3$	M1A1	For $\frac{1}{2}x + \frac{1}{6}x^2 - \frac{1}{18}x^3$
	$= 3\pi$	A1 **5**	
(iii)	When $x = 4$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{4}$	B1	
	Unit normal vector is $\begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$	M1 A1 ft	Finding a normal vector Correct unit normal (either direction)
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{4}x^{-3/2} - \frac{1}{4}x^{-1/2} \quad (= -\frac{5}{32})$	B1	
	$\rho = \frac{\{1 + (-\frac{3}{4})^2\}^{3/2}}{(-)\frac{5}{32}} \quad (= \frac{125/64}{5/32} = \frac{25}{2})$	M1 A1 ft	Applying formula for ρ or κ
	$\mathbf{c} = \begin{pmatrix} 4 \\ -\frac{2}{3} \end{pmatrix} + \frac{25}{2}\begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$	M1	
	$= \begin{pmatrix} -3\frac{1}{2} \\ -10\frac{2}{3} \end{pmatrix}$	A1 A1 **9**	
(iv)	Differentiating partially w.r.t. p	M1	
	$0 = 2p\,x^{1/2} - p^2 x^{3/2}$	A1	
	$p = \frac{2}{x}$		
	Envelope is $y = \frac{4}{x^2}x^{1/2} - \frac{1}{3}\frac{8}{x^3}x^{3/2}$	M1 A1	
	$y = \frac{4}{3}x^{-3/2}$	A1 **5**	

4 (i)	$\mathrm{s\,t}(x)=\mathrm{s}\left(\frac{x}{x-1}\right)=\frac{\frac{x}{x-1}-1}{\frac{x}{x-1}}$	M1	
	$=\frac{x-(x-1)}{x}=\frac{1}{x}=\mathrm{r}(x)$	A1 (ag)	
	$\mathrm{t\,s}(x)=\mathrm{t}\left(\frac{x-1}{x}\right)=\frac{\frac{x-1}{x}}{\frac{x-1}{x}-1}$	M1	
	$=\frac{x-1}{(x-1)-x}=1-x=\mathrm{q}(x)$	A1 **4**	
(ii)	see table below	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iii)	Element: p q r s t u; Inverse: p q r u t s	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iv)	{ p }, F { p, q }, { p, r }, { p, t } { p, s, u }	B1B1B1 B1 **4**	*Ignore these in the marking* Deduct one mark for each non-trivial subgroup in excess of four
(v)	Element: 1, -1, $e^{\frac{\pi}{3}j}$, $e^{-\frac{\pi}{3}j}$, $e^{\frac{2\pi}{3}j}$, $e^{-\frac{2\pi}{3}j}$; Order: 1, 2, 6, 6, 3, 3	B4 **4**	Give B3 for 4 correct, B2 for 3 correct B1 for 2 correct
(vi)	$2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=13,\ 2^6=\mathbf{7}$ $2^7=14,\ 2^8=9,\ 2^9=\mathbf{18},\ 2^{10}=17,\ 2^{11}=15,\ 2^{12}=11$ $2^{13}=3,\ 2^{14}=6,\ 2^{15}=12,\ 2^{16}=5,\ 2^{17}=10,\ 2^{18}=1$ Hence 2 has order 18	M1 A1 A1 **3**	Finding (at least two) powers of 2 For $2^6=7$ and $2^9=18$ Correctly shown *All powers listed implies final A1*
(vii)	G is abelian (so all its subgroups are abelian) F is not abelian	B1 **1**	*Can have ‘cyclic’ instead of ‘abelian’*
(viii)	Subgroup of order 6 is $\{1,\ 2^3,\ 2^6,\ 2^9,\ 2^{12},\ 2^{15}\}$ i.e. $\{1,\ 7,\ 8,\ 11,\ 12,\ 18\}$	M1 A1 **2**	 or B2

Table for (ii):

	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	u	p	t	s	q
s	s	t	q	u	r	p
t	t	s	u	**q**	**p**	**r**
u	u	r	t	**p**	**q**	**s**

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.16 & 0.28 & 0.43 & 1\\ 0.84 & 0 & 0 & 0\\ 0 & 0.72 & 0 & 0\\ 0 & 0 & 0.57 & 0\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two columns correct
(ii)	$\mathbf{P}^9\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.3349\\0.3243\\0.2231\\0.1177\end{pmatrix}$ Prob(C) = 0.2231	M2 A1 **3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $\mathbf{P}^4\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.5020\\0.2851\\0.1577\\0.0552\end{pmatrix}$	B1 M1 A1 **3**	First column of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7=\begin{pmatrix}. & . & . & .\\ 0.2869 & . & . & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ $\mathbf{P}^8=\begin{pmatrix}. & . & . & .\\ . & . & . & .\\ . & 0.2262 & . & .\\ . & . & . & .\end{pmatrix}$ Probability is 0.2869×0.2262 $=0.0649$	M1M1 M1 A1 **4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16}=1.19$ (3 sf)	M1 A1 **2**	Allow 1.2
(vi)	$\mathbf{P}^n \to \begin{pmatrix}0.3585 & 0.3585 & 0.3585 & 0.3585\\ 0.3011 & 0.3011 & 0.3011 & 0.3011\\ 0.2168 & 0.2168 & 0.2168 & 0.2168\\ 0.1236 & 0.1236 & 0.1236 & 0.1236\end{pmatrix}$ A: 0.3585 B: 0.3011 C: 0.2168 D: 0.1236	M1 M1 A2 **4**	Evaluating $\mathbf{P}^n$ with $n\ge 10$ *or Obtaining (at least) 3 equations from* $\mathbf{Pp}=\mathbf{p}$ Limiting matrix with equal columns *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft **2**	
(viii)	$\begin{pmatrix}a & b & c & 1\\ 1-a & 0 & 0 & 0\\ 0 & 1-b & 0 & 0\\ 0 & 0 & 1-c & 0\end{pmatrix}\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}=\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ $0.4a+0.25b+0.2c+0.15=0.4$ $0.4(1-a)=0.25$ $0.25(1-b)=0.2$ $0.2(1-c)=0.15$ $a=0.375,\quad b=0.2,\quad c=0.25$	M1 A1 M1 A1 **4**	Transition matrix and $\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ Forming at least one equation *Dependent on previous M1*

Post-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 0.16 & 0.84 & 0 & 0 \\ 0.28 & 0 & 0.72 & 0 \\ 0.43 & 0 & 0 & 0.57 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	B2	**2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two rows correct
(ii)	$\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \mathbf{P}^9$ $= \begin{pmatrix} 0.3349 & 0.3243 & 0.2231 & 0.1177 \end{pmatrix}$ $\text{Prob}(C) = 0.2231$	M2 A1	**3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \mathbf{P}^4$ $= \begin{pmatrix} 0.5020 & 0.2851 & 0.1577 & 0.0552 \end{pmatrix}$	B1 M1 A1	**3**	First row of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7 = \begin{pmatrix} . & 0.2869 & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{pmatrix}$ $\mathbf{P}^8 = \begin{pmatrix} . & . & . & . \\ . & . & 0.2262 & . \\ . & . & . & . \\ . & . & . & . \end{pmatrix}$ Probability is 0.2869×0.2262 $= 0.0649$	M1M1 M1 A1	**4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16} = 1.19$ (3 sf)	M1 A1	**2**	Allow 1.2
(vi)	$\mathbf{P}^n \to \begin{pmatrix} 0.3585 & 0.3011 & 0.2168 & 0.1236 \\ 0.3585 & 0.3011 & 0.2168 & 0.1236 \\ 0.3585 & 0.3011 & 0.2168 & 0.1236 \\ 0.3585 & 0.3011 & 0.2168 & 0.1236 \end{pmatrix}$ A: 0.3585 B: 0.3011 C: 0.2168 D: 0.1236	M1 M1 A2	**4**	Evaluating $\mathbf{P}^n$ with $n \geq 10$ *or Obtaining (at least) 3 equations from* $\mathbf{p}\mathbf{P} = \mathbf{p}$ Limiting matrix with equal rows *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft	**2**	
(viii)	$\begin{pmatrix} 0.4 & 0.25 & 0.2 & 0.15 \end{pmatrix} \begin{pmatrix} a & 1-a & 0 & 0 \\ b & 0 & 1-b & 0 \\ c & 0 & 0 & 1-c \\ 1 & 0 & 0 & 0 \end{pmatrix}$ $= \begin{pmatrix} 0.4 & 0.25 & 0.2 & 0.15 \end{pmatrix}$ $0.4a + 0.25b + 0.2c + 0.15 = 0.4$ $0.4(1-a) = 0.25$ $0.25(1-b) = 0.2$ $0.2(1-c) = 0.15$ $a = 0.375,\quad b = 0.2,\quad c = 0.25$	M1 A1 M1 A1	**4**	Transition matrix and $\begin{pmatrix} 0.4 & 0.25 & 0.2 & 0.15 \end{pmatrix}$ Forming at least one equation *Dependent on previous M1*